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ABSTRACT

Definitions for the meaning of the words "classical" and "quantum" are introduced, which emphasize that the essential difference between classical and quantum theories is the role of the measurement. This is most clearly exemplified in the operational definition of space-time; physical space is not a threefold product of the real line with itself. Also, the role of matter in defining space-time is pointed out. Finally, it follows from our arguments that there can be no "hidden variables", in terms of which physics would be deterministic.

P.S. Any comments are welcome.

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"CLASSICAL", "QUANTUM", AND THE STRUCTURE OF SPACE-TIME

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The purpose of any theory in physics is to describe a whole class of observations in a unified manner. One tries to reduce the individuality and variations of the observations to the variation of just a few parameters. The language in which a theory is expressed, as well as the tool used to manipulate the theory, is mathematics; theoretical physics constructs an image of physical reality within the realm of mathematics. Such an image or model is what we retain in our memory, since it is hardly feasible and very uneconomical to remember a mass of unrelated observations, and it is through such a model that we "understand" Nature. However, the model is not reality, and it only describes reality in so far as its predictions agree with experimental results.

Let us first of all consider what we mean by reality. The physical world, the subject matter treated by physics, is that which we, as human beings, perceive; it is the sum total of all our measurements and observations. It is an intrinsic feature of our nature that space appears to be three-dimensional; with our senses we can relate the position of a material point object relative to an other by means of three numbers. Space is the relation between objects, as perceived by us.

The concept of time is even more firmly rooted in our nature: Without memory there could be no such thing as time. Memory gives a new dimension to our perception; not only can we observe the spatial relations between objects, but we can observe that these relations change. The concepts of spatial relations between objects and change in these relations are inseparable from our human nature, and are intuitively known to us in the sense of Kant's synthetical knowledge a priori. - Just as space is the relation between objects, time is the relation between changes in spatial relations. As a reference change we take a periodic motion, where periodic simply means that, as far as we are able to perceive, the motion repeats itself in all detail. The time for another change to take place is then the number of cycles of the reference motion coinciding with the change. - Of course, at this point one has to be a bit more careful about the definition of simultaneity at spatially separated points, etc., but those arguments are well known from special relativity.

So far, we have seen that all our observations, i.e., the physical world, can be described in a four-dimensional framework. But let us look a little closer at these observations. Any observation or measurement involves an interaction between the human and the object of interest, possibly through a measuring instrument. Interaction means a transfer of energy from the object to us, thus a change is involved, so that all our observations are necessarily four-dimensional in nature. It is true, of course, that in most instances the change is imperceptible, as when we look at an object like a table, so that three-dimensional space, completely divorced from time, makes good sense to us. But it is nevertheless important to realize that, strictly speaking, a three-dimensional, static world has no physical reality, since it can not be perceived.

Now, one might imagine that the above case of imperceptible change due to the interaction could always be brought about by making the measurement so that the interaction is arbitrarily small. But we shall shortly see how the experimental evidence in fact excludes this possibility.

Since Nature is what it is irrespective of our labels or efforts, such labels as "classical" or "quantum" must of course refer to the model. We now introduce the following definition:

Definition A. By a classical theory or model we understand any description of the physical world directly in terms of the observable results of measurements.

To remind ourselves of this connection between the meaning of "classical" and the results of measurements, we shall sometimes refer to the latter as c-numbers.

By this definition it is perfectly possible to have Planck's constant, h, appear in a classical theory. And on the surface it seems that introducing the quantum of electromagnetic energy, with $E = h\nu$, would still leave the theory classical, since $E = h\nu$ is an experimental fact. However, it is clear from the foregoing that any classical theory exists only with respect to our four-dimensional reference frame, and we now show that the introduction of the relation $E = h\nu$, together with the special theory of relativity, causes a very significant change in the structure of space-time. The assumption implicit in all of theoretical physics, namely that four-dimensional space-time has (at least locally) the structure of a four-fold direct product of the real line with itself, is no longer correct.

Consider now a system, which we for definiteness shall call a particle, with mass m_o. We want to localize it in space-time by means of letting it interact with a photon, and in particular we want to localize it in time to within an interval Δt. This means that we must, essentially, use a photon of frequency $\nu = (\Delta t)^{-1}$ to interact with the particle, since we measure time by counting periods. So the momentum of the photon is given by $p = h/c\ \Delta t$. The momentum transferred to the particle by the interaction is of the same order of magnitude (see e.g., Schiff, "Quantum Mechanics," p. 9 for a description of the type of measurement we are using here). At the end of the interaction the particle has gained a velocity $v = p/m$, so that during the interaction the average velocity was $p/2m$. The time at which the interaction took place cannot be determined more accurately than to within Δt, again simply because our time-scale stops at Δt if we use a photon of frequency $(\Delta t)^{-1}$. Therefore, in the time the interaction takes place, the particle has moved a distance

$$\Delta x = p\Delta t/2m_0 = h\Delta t/2m_0 c\Delta t = h/2m_0 c \ .$$

If the particle has an initial velocity v_0, there would, of course, be an additional uncertainty of $\Delta t v_0$, but this could, in principle, be made arbitrarily small by making Δt arbitrarily small. - If v_0/c is not small compared to unity, then the above Δx, which was essentially calculated in the rest frame of the particle, is multiplied by $\{1 = (v_0/c)^2\}^{1/2}$, i.e., just the ordinary relativistic contraction. The length Δx, defined within one inertial frame, is of course independent of the particular inertial frame. - Instead of a photon we could have used a massive particle to interact with the original particle. However, since we have the relation $p = h\nu/v$ (e.g., from electron diffraction), it is clear that, for a given frequency, the least momentum will result if v is as large as possible, which according to the special theory of relativity is c.

The very important thing is that the result

$$\Delta x = h/2m_0 c \tag{A}$$

shows that this uncertainty in the position does not depend on the frequency of the photon. In other words, the inherent space-time nature of our perception limits our resolving power in three-dimensional space.

That the mass appears in the result is not surprising, since physical space is the relation between material objects. Space and time are two inseparable aspects of the material world, or, conversely, space and time are connected through matter. This is the first fundamental difference between space-time and a four-dimensional linear vector space; the four dimensions are not completely independent of each other.

The second fundamental difference is, of course, the fact that three of the four dimensions, the spatial ones, are not defined with arbitrary high accuracy. - Let us see what uncertainty our result A predicts for some common particles. For a proton, $\Delta x \approx 10^{-15}$m, for an electron $\Delta x \approx 2 \cdot 10^{-12}$m. It is clear that these answers are only approximate, since the result A is itself only a rough order-of-magnitude estimate.

These distances are quite small; however, the existence of this basic uncertainty in the definition of physical space can easily be shown to imply the familiar Heisenberg uncertainty relation. Consider again a particle of mass m at rest, then $\Delta p = m dx/dt = m\Delta x/\Delta t$, and taking the limiting value of Δx from result A, we obtain $\Delta p = h/2c\Delta t$. The uncertainty in position due to the finite wavelength of the photon (see Schiff) is $\Delta x' \approx \lambda$. - We note that this $\Delta x'$ has nothing to do with the Δx of result A, the present $\Delta x'$ can be made arbitrarily small by letting $\lambda \to 0$. - This leads to $\Delta x' \cdot \Delta p \approx h/2\nu\Delta t = h/2$, which is the usual result. Of course, $\Delta x' \geq \Delta x$ if we want to interpret x' as the measurable position of the particle. It is only the usual, but from our standpoint unwarranted, identification of real space with R^3 which allows $\Delta x' \to 0$. At any rate, except for this limitation, the Heisenberg uncertainty principle is a consequence of result A, and we may therefore view the limitation on the definition of physical space as the basic reason for the probabilistic description of Nature which is incorporated into quantum mechanics.

Since all measurements are described in a space-time frame due to the nature of our perception, it is clear that, as a result of the uncertainty in the definition of space, <u>all classical variables are random variables</u>. The distribution of the random variable depends on the mass of the system which the variable is describing. A nonstatistical theory of Nature can therefore obviously not be constructed <u>directly</u> in terms of c-numbers. A physical system must be represented in the theory by an abstract (i.e., not measurable) "something," such that when we specify a particular measurement, there will be a corresponding operation in the theory which, when applied to the "something," will give the observed random variable. We accordingly introduce the definition:

Definition B. By a quantum theory or model we understand our description of the physical world in terms of abstract, non-measurable quantities, from which the random c-numbers (i.e., their distributions) are extracted by specifying the appropriate measurement. A quantum theory must thus contain a theory of measurements; for each physical measurement it must specify what the corresponding operation on the abstract model consists of.

This feature of quantum theories, that the c-numbers do not emerge directly, but only after a particular experimental situation has been defined, leads, as is well known, to two complementary classical descriptions of any given system; this is the so-called wave-particle duality. However, it must be emphasized that the wave-particle duality is essentially different for systems with rest mass and for systems without rest mass. In the former case it is the particle aspects (i.e., size, mass, position) which are directly measurable, whereas the wave aspects (i.e., frequency, phase) are only inferred from other effects. The wave associated with a massive particle is not a physical quantity, it is a result of the randomness of the c-numbers, and becomes important only when this randomness becomes significant.

In the case of systems without rest mass, say light, it is the wave aspects which are directly measurable, and the particle aspects are inferred. For a photon the wave-particle duality is complete in the sense that both are always needed for a description of the photon. The fact that a wave description of the electromagnetic field is often adequate arises because the electromagnetic field often contains so many photons that effects relating to the particle aspect of the photon gets "washed out," and has nothing to do with the size of Δx relative to distances of interest. The length Δx is always infinite for photons, a free photon has no location in space, nor any size. In other words, we can say that the two concepts particle and wave, and the resulting duality, exists only due to our artificial (but usually very convenient and useful) separation of massive systems and their interactions. In reality they are always combined.

Our arguments were based on the assumption that the speed of light is the maximum speed at which energy can propagate and on de Broglie's relation, both of which are uncontested by present experimental evidence. It seems then, that unless new experimental results should appear that would limit the universal validity of the two assumptions mentioned, the limitation on the definition of physical space is an inescapable consequence of the space-time nature of our perception. In particular, it follows that any "hidden variables," in terms of which a c-number description of Nature would be deterministic, would have to be in a conceptually different framework; i.e., not in physical space-time. Such variables would, however, not belong to what we now call physics.