The Concept of Emergence, and a Simple Example

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Abstract. Emergence is an important aspect of the system concept, and its relevance and application to systems engineering continues to be a subject of debate. The paper gives a number of examples of emergent properties, and then describes a very simple system that exhibits an emergent property that is defined in terms of the interaction between the elements and the structure of the system defined by the interaction.

1 Introduction

1.1 Emergence

Emergence is a central feature of the system concept, and consequently also a significant aspect of the systems engineering methodology. While there are numerous definitions of emergence, with different emphasis and within different contexts [1], it is often summed up by the statement that "the whole is more than the sum of its parts". That is, the system has properties that are not evident in any of its elements. Or, from another perspective, the properties of the system are determined not only by the properties of the elements, but also by the interactions between them. Most often, at least in the context of engineering, the properties of interest are in the form of behaviour or capabilities; that is, the system has capabilities that are not found in any of its elements.

Examples of emergence are everywhere, and in the next subsection we shall look at a number of them. However, while it is easy to find examples, they all give slightly varying perspectives or understanding of the concept, and this explains why there is still considerable discussion about exactly what emergence is [2]. We need a common understanding, or framework, that encompasses all these different views of emergence, and to that end we recall two important aspects of the system concept. The first is that a system is a *mode of description* [3]; the same entity can always be described as a system, consisting of interacting elements, or as a "black box", i.e. as a single element. In the latter mode, the properties of the single element are just that, and there can be no distinction between emergent and non-emergent properties; in the former mode we can identify those properties as emergent that are not present in any of the elements. That is, from this point of view, emergence is simply a result of describing an entity as composed of interacting elements. The transition between these two modes of description goes both ways; that is, we can start out with a single, complex entity and through a process of analysis end up with a description in terms of less complex, but interacting entities, or start out with a set of entities and through a process of synthesis, i.e. allowing them to interact, end up with a description of a single entity with complex features.

The second aspect is that the interactions are inherent in the elements; that is, the elements always have the *ability* to interact. If we consider a single element, this ability is then what we can observe about the element as a "black box"; when we form elements into systems, some (or all) of this ability is utilised as internal interactions and some as external interactions. Therefore, what we can observe about a system from the outside (i.e. considering the system as a single entity) can always be described in terms of the parameters describing the elements; nothing is added to what is already present in the elements, and the interactions are purely formal or *logical* in the sense that they have no properties of their own [4]. It may be very *convenient* to introduce new, system-related parameters, such as the pressure or temperature of a gas, but these system parameters can always be expressed in terms of the element parameters.

1.2 Examples

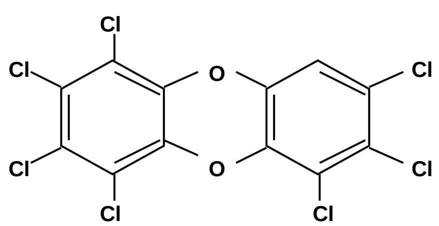
The following examples, offered in no particular order, serve to illustrate some of the issues that arise in providing a general definition of emergence.

a. *Electromagnetic radiation*. Atoms (and molecules) emit electromagnetic radiation when they change their state from one of higher energy to one of lower energy. A collection of atoms will emit incoherent radiation, but if the interaction between them is increased, the interaction becomes coherent and the resultant system is then called a laser. One might say that the coherent radiation is an emergent property of the set of interacting atoms, but in forming a laser, we have introduced additional elements, e.g. the mirrors. Coherence is not quite an instance of emergence; it involves a global coordination rather than just an interaction between individual elements.

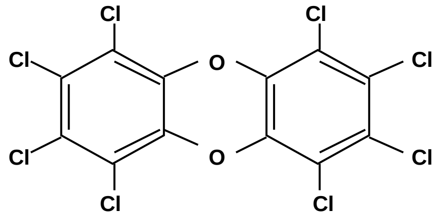
b. *Chemical synthesis*. One of the fundamental processes in chemical engineering is reaction, in which two or more substances are brought together under controlled conditions to form one or more new substances, and where the new substances have properties that are not found in any of the components. There is now a vast amount of knowledge about chemical reactions that allows a degree of predictability, but the unpredictability of the properties of new substances is particularly striking when it comes to their interactions with living matter, which, by the way, accounts for the cost of developing pharmaceuticals, and two examples illustrate this. The first is two members of a class of substances call dioxins, and the emergent property is the Toxic Equivalent Factor (TEF), a measure of the carciogenic effect of the substance. These two substances, with structures as shown in Fig. 1, differ only by a single chlorine atom, but the TEF differs by a factor of 100 [5]. - As an aside, imagine this case translated into the realm of teams, with people instead of atoms. Could the addition of a single person to a team of 21 persons result in a change in effectiveness of a factor of 100?

   The second example is a herbicide sold under the trade name of Dual (a registered trademark of Syngenta Agro AG, Switzerland). Here, the difference between the two substances is not even a difference in the atoms making up the molecule, but simply a difference in the structure of the molecule (i.e. they are isomers). The structures of the two isomers resulting from a stereoselective reaction are shown in Fig. 2, and the herbicide effectiveness, measured at an application density of 500 grams per hectare, is 92 % for the S isomer, whereas that of the R isomer is only 39 % [6].

   In both of these cases, there is nothing added to the set of molecules; it is simply a matter of turning on the interaction (in the first case) or of changing the interactions (in the second case), and as such they are both proper illustrations of emergence. However, given a molecule of each of these substances and their external interactions (which are very complex), it would be impossible to deduce their composition as systems of atoms and to classify any part of their behaviour as emergent.



(a) Heptachlorodibenzodioxin, TEF = 0.01



(b) Octachlorodibenzodioxin, TEF = 0.0001

Figure 1 Two members of the dioxin family.

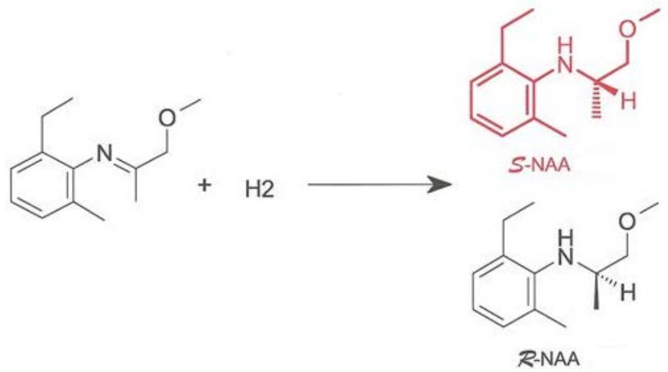


Figure 2 The stereoselective reaction in the S-NAA-process.

c. *Language*. In order to express something complex, such as a story, we use a limited set of vowels that can be combined to form words, the words are subdivided into groups (nouns, verbs, predicates, etc.) and combined to form sentences, and the whole story is a string of sentences. However, the elements need to *interact*, and in the case of language the interaction takes place in the mind of the listener and is determined by the sequence of the vowels, words, and sentences in accordance with the rules of grammar. The meaning of a sentence emerges from the set of words, and the story emerges from the set of sentences, but again, for these elements to interact and form a system, it takes something additional to the elements – the brain.

d. *Synergy*. This is the increased performance of a group of people engaged on performing a task when they interact rather than working separately. The increased performance emerges as a result of the interaction that forms them into a system, and as people inherently have the ability to interact, nothing additional is required. However, as we all know, that while we have this ability, we may not always use it very effectively, and it takes a manager, moderator, facilitator, or the like to make the interaction effective.

e. *Electronic circuit*. An integrated circuit might perhaps be the best example of how a set of simple elements can have a very complex capability when the are allowed to interact and form a system. But even a very simple system, consisting of only two elements, a capacitor and an inductor, exhibits a behaviour, resonance, that is not present in either of the elements; it emerges from the interaction that forms the two components into a system.

f. *Colour*. Any colour can emerge by combining the three primary colours, red, blue, and green, in the right proportions.

These examples each illustrate a slightly different meaning or content of the concept of emergence. In particular, there is considerable variation in the extent to which the emergent property is due to the difference in the properties of the elements, such as in the last example, where it is not surprising that combining two different colours results in a new colour. What we are really interested in are cases where properties of the system depend on the *manner* in which the elements interact, i.e. on the *structure* of the system. This was illustrated by the examples from chemistry, and it is not difficult to find examples of such systems of interest to systems engineering. However, they are usually quite complex and mostly involve humans as system elements, which makes a detailed, quantitative description difficult. What we would like to have are some examples of simple systems of identical elements, where a system property is demonstrably dependent on the interaction being present and on the structure of the system formed by the interaction. One such example is presented in the next section.

2 The System

2.1 The System Elements

The system consists of *n* indistinguishable *production elements*, existing in a common environment. Each element has a fixed *production rate* of one unit per time period, and a fixed *production cost*, *c*, measured in dollars per unit. The production elements sell their products into a common market, where they fetch an average price, *h*, measured in dollars per unit, given by

$$\bar{h} = \frac{a}{n+b} \, , \tag{2.1}$$

where *a* [$/period] and *b* [units/period] are constants characterising the price elasticity of the market. (Note that in this model, the $ sign does not necessarily indicate the US dollar; it is simply a unit of value.) It is the interaction of the elements through this common market that makes them form a system, albeit that this is an indirect interaction. A proper, direct interaction will be introduced in the next section.

However, each individual element experiences a random fluctuation in the price obtained, so that the price obtained by an element in any one period, *h*, is given by

$$h = \frac{a}{n+b}(1+x) \, ; \tag{2.2}$$

where *x* is a random variable that is equally likely to take on a value anywhere in the interval ±X. The variance of this distribution is $X^2/3$.

In one time period, an element generates a *surplus*, *u*, given by

$$u = \left(\frac{a}{n+b}(1+x) - c \right). \tag{2.3}$$

This surplus, which may become negative, is added to an *accumulator* with capacity S [$]; the current value of the amount stored in the accumulator is denoted by *s*. As long as the accumulator is full, i.e. *s* = S, any further positive surplus is discarded. However, if the accumulator is emptied, i.e. *s* = 0, the element ceases to exist, and the value of *n* decreases by 1.

2.2 System Behaviour

Conceptually, the behaviour of this system is quite simple. If the initial value of *n*, n_0, is greater than a certain value, the average surplus will be negative, and whatever the initial value of *s* is, individual values of *s* will become zero after some time and elements will cease to exist until the number of elements has been reduced to a sustainable value. The value of *n* at which the average value of the surplus becomes zero might be called the *break-even population*, n_a, given by $n_a = a/c - b$. The value of *n* at which no further reduction is possible, even in an infinitely long time period, n_∞, is given by

$$n_\infty = \frac{a}{c}(1-X) - b; \tag{2.4}$$

rounded down to the next integer value, and as the smallest value of n_∞ is 1 (a monopoly), we obtain the following constraint,

$$\frac{a}{c}(1-X) - b \geq 1. \tag{2.5}$$

We might consider n_∞ to be the *sustainable population*; however, within a practical time-frame, the steady-state population will lie somewhere between n_a and n_∞.

In order to get a more detailed understanding of this behaviour, let the value of *i* indicate the end of the i-th time period, and let each element have the same initial condition, $s = s_0$. Then, after *i* time periods, but before any element has been discarded, so that $n = n_0$, and using the central limit theorem, the value of *s* for each element can be approximated by the expression

$$s(i) = s_0 + i \cdot h(n_0) + \sqrt{i} \cdot q(n_0) \cdot z; \tag{2.6}$$

where

$$h(n_0) = \frac{a}{n_0 + b} - c; \tag{2.7}$$

$$q(n_0) = \frac{a}{n_0 + b} \cdot \frac{X}{\sqrt{3}}; \tag{2.8}$$

and *z* is a random variable with the standard normal probability density,

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \tag{2.9}$$

The first element is likely to be discarded when the probability of *s* becoming zero (or less) is about $1/n_0$, or

$$s_0 + i \cdot h(n_0) + \sqrt{i} \cdot q(n_0) \cdot d = 0; \tag{2.10}$$

with *d* defined by

$$\frac{1}{n_0} = \int_{-\infty}^{d} f(z)dz. \tag{2.11}$$

From a Table of the standard normal distribution, we find the following values of *d* as a function of n_0,

n_0	5	10	20	30	100
d	-0.84	-1.28	-1.65	-1.83	-2.33

which, for our purposes and with an rms error of 6 %, can be approximated by the expression

$$d = -1.7 - 0.007n_0 + \frac{4}{n_0}\,. \tag{2.12}$$

For given values of *a*, *b*, *c*, and n_0, we can then determine the number of steps until the first element is likely to be eliminated. For example, with *a* = 2000, *b* = 5, *c* = 60, X=0.2, and n_0 = 30, we have *h* = -2.857 and q = 6.6, so that Eq. (2.11) becomes

$$s_0 - 2.857 \cdot i - 6.6 \cdot 1.83\sqrt{i} = 0; \tag{2.13}$$

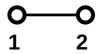
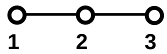
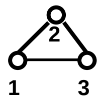
and, with s_0 = S/2 = 500, the solution is *i* = 127.

This can be compared with a step-by-step simulation, using the random number generator provided in Excel, which after 500 runs gives an average of *i* = 123 with a standard deviation of 11. For these same parameter values, n_a = 28 and n_∞ = 22, and the simulation gives a steady-state (after 10,000 time steps) value of 27 ±1, which is reached after typically 200 time steps. That is to say, that the probability of any one of the elements being eliminated is 3/30 = 0.1.

3 Direct Interaction

3.1 The Interaction

We now introduce a direct interaction between some of the elements, so that these form a distinct subsystem within the population. The nature of the interaction is that an element can share its accumulator with up to two other elements, so that if we number the elements and consider the first three, there are three possible, distinct subsets:

	*s(1)* = *s(2)* = (*s(1)* + *s(2)*)/2
	*s(2)* = (*s(1)* + *s(2)* + *s(3)*)/3 ; *s(1)* = (*s(1)* + *s(2)*)/2 ; *s(3)* = (*s(2)* + *s(3)*)/2
	*s(1)* = *s(2)* = *s(3)* = (*s(1)* + *s(2)* + *s(3)*)/3

The expressions in the right-hand column are the conversions of the three accumulator values following the completion of a time step for all *n* elements, but before the elimination of elements with $s \leq 0$.

3.2 Subsystem Behaviour

To demonstrate the effect of the interaction, the simulation was again run 500 times, but this time with the following parameter values: *a* = 2000, *b* = 5, *c* = 70, X = 0.2, n_0 = 30, and s_0 = S/2 = 500. The values of n_a and n_∞ are now 24 and 18, respectively, and the elimination probability without any direct interaction is 29.2 % (corresponding to a steady-state population of about 21).

With direct interaction and the formation of a subsystem, the emergent property of that subsystem is the reduced elimination probability of its elements. In the case of the first of the three subsystems defined above, the failure rates of the two subsystem elements are reduced to 20.8 % each (with a standard deviation of about 2 %).

In the second case, the linear structure, elements 1 and 3 have an elimination probability of 16.5 %, and element 2 one of 14.8 %.

And, finally, in the third case, where the coupling between the subsystem elements is the closest, the elimination probability of each element is 14.4 %.

4 Discussion

The results obtained in the previous section are not surprising; the benefit of pooling resources is well known and exploited in applications ranging from company mergers to electricity grid interconnections. The purpose of this example is to simplify the situation to the extent that we can obtain a single, quantitative measure of the effect of the interaction on the behaviour of the system. In addition, it also demonstrates the importance of fluctuations; working only with averages would not allow us to determine a steady state. The increasing importance of fluctuations in engineering design and the relationship to *risk* was discussed in [7], where it was pointed out that this requires a problem formulation in terms of distributions rather than scalar variables.

References

1. There is a large literature on the subject of emergence, and general references can be found on the web, e.g. at http://en.wikipedia.org/wiki/Emergence or http://plato.stanford.edu/entries/properties-emergent/. Much of the discussion around emergence is of a philosophical nature and not directly relevant to engineering (which is more down-to-earth), but an interesting article, and one with a large bibliography, is one by Bickhard, M.H. and D.T. Campbell, available at http://www.lehigh.edu/~mhb0/emergence.html.
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